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CH. 1 - ROTATIONAL DYNAMICS

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Introduction

circular motion → Circular motion is a movement of an object along the circumference of a circle or rotation along a circular path.

eg. i) Earth revolve around the sun. ii) Electron revolve around nucleus.

Characteristics of Circular motion

1. It is an accelerated motion : As the direction of velocity changes at every instant, it is an accelerated motion.

2. It is a periodic motion : During the motion, the particles repeats its path along the same trajectory. Thus, the motion is periodic.

Kinematics of Circular motion

	Linear	Angular
displacement	s	θ
velocity	$v = \frac{ds}{dt}$	$\omega = \frac{d\theta}{dt}$
Acceln	$a = \frac{dv}{dt}$	$\alpha = \frac{d\omega}{dt}$

Relation between linear and angular term ⇒

$s = r\theta$

$v = r\omega$

$a = r\alpha$

→ Axial vector

$$\alpha = \frac{d\omega}{dt} = \frac{\omega_2 - \omega_1}{t}$$

$T = \frac{2\pi}{\omega}$

But, $\boxed{\omega = 2\pi n}$ or $\boxed{\omega = \frac{2\pi}{T}}$

Relation betn angular velocity & Frequency.

↳ Frequency.

$$\alpha = \frac{2\pi n_2 - 2\pi n_1}{t}$$

$$\boxed{\alpha = \frac{2\pi (n_2 - n_1)}{t}}$$

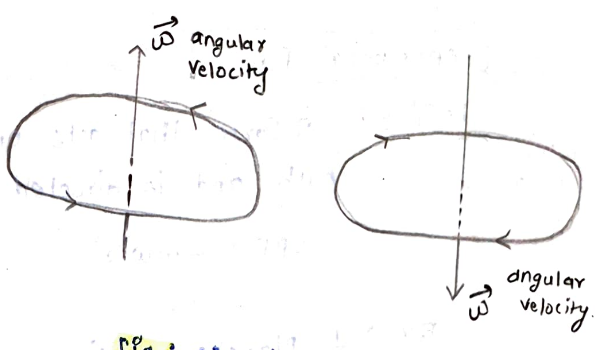


Fig: Direction of Angular velocity.

Period: (T) The time taken by particle performing UCM to complete one revolution is called period.

Frequency: (n) The no. of revolutions performed by particle performing UCM in unit time is called as Frequency of revolution.

↳ Reciprocal of period.

mcq ⇒ circular motion में acceleration constant नहीं होता।

Uniform circular motion

acceleration ≠ 0

(ω) → velocity

Define ⇒ During circular motion if the speed (only magnitude) of the particle remains constant, it is called UCM.

centripetal acceleration

Only the direction of its velocity changes at every instant in such a way that the velocity is always tangential to the path.

$$\vec{a} = -\omega^2 \vec{r}$$ → vector form.

$$a = \frac{v^2}{r} = \omega^2 r = v\omega$$

↳ centripetal accln or radial acceln

⊙ out ward

⊗ In ward

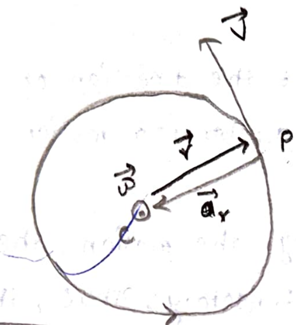


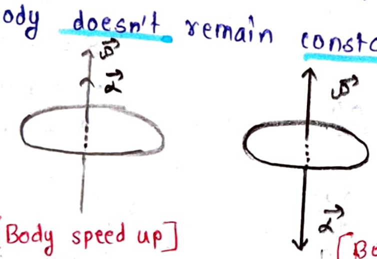
fig: Direction of linear velocity & acceleration

- The accln of particle performing UCM is directed towards the centre and along the radius of circular path is called as centripetal accln.

Non-Uniform circular motion

Define ⇒ Non-UCM is a circular motion when the speed of the body doesn't remain constant.

eg: fan on.



[Body speed up]

[Body speed down]

eg: fan off.

fig: Direction of angular acceleration.

Centripetal Force

2m

Characteristics. (next pg.)

Define ⇒ A Force that acts on a body moving in a circular path and is directed towards the centre of the circle.

किसीके कारण तैयार होता है. (original force नहीं है.)

$$CPF = -m\omega^2 \vec{r}$$ ↳ Vector form

$$CPF = \frac{mv^2}{r} = mr\omega^2 = mv\omega$$ ↳ Magnitude form

Ex ⇒ 1. Planets orbiting around the sun.

2. A car moving along a curve road.

Tangential acceleration:

- The accln responsible for changing the magnitude of velocity is directed along or opposite to the velocity, hence always tangential and is called as tangential acceleration. $[\vec{a}_T]$.

$$a_T = r\alpha$$

Centrifugal Force

Define ⇒ Centrifugal Force is a pseudo Force in a circular motion which acts along the radius and is directed away from the centre of the circle.

$$\boxed{CFF = +m\vec{r}\omega^2}$$
↳ vector form.

$$\boxed{CFF = \frac{Mv^2}{r} = Mr\omega^2 = mv\omega}$$
↳ In magnitude

Ex ⇒
1. weight of an object at the poles and on the equator.
2. A bike making a turn.
3. Vehicle driving around a curve.

Characteristics of centrifugal force:
- always directed away from the centre
- it is not a real force.
- experienced in the

Applications of UCM

Imp ① **Vehicle along a Horizontal circular Track :-** [2m]

$N = Mg$ — ①

$F_s = \frac{Mv^2}{r}$ — ②

② by ①

$\frac{F_s}{N} = \frac{\cancel{M}v^2/r}{\cancel{m}g}$

$\frac{F_s}{N} = \frac{v^2}{rg}$

but,

$$\boxed{\mu_s = \frac{F_s}{N}}$$

$\mu_s = \frac{v^2}{rg}$

$v^2 = \mu_s r g$

$$\boxed{V_{max} = \sqrt{\mu_s r g}}$$

where, μ_s = coefficient of friction → unitless and dimensionless quantity.
r = Radius of curve road.

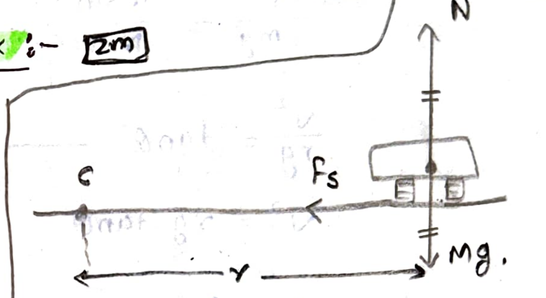


Fig: vehicle on horizontal road.

② **Well (or wall) of Death (मौत का कुआँ) :-**

$N = \frac{Mv^2}{r}$ — ①

$F_s = Mg$ — ②

$F_s \le \mu_s N$

$Mg = \mu_s \frac{\cancel{M}v^2}{r}$

$\frac{rg}{\mu_s} = v^2$

$$\boxed{V_{min} = \sqrt{\frac{rg}{\mu_s}}}$$

i) Normal reaⁿ N acting horizontally & toward the centre
ii) Weight mg, acting vertically downword.
iii) Force of static friction F_s, acting vertically upwards between vertical wall and the tyres.

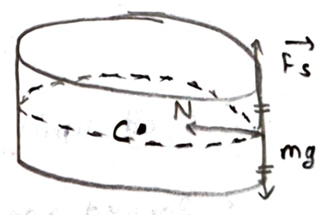


Fig: well of death

③ Vehicle on a Banked Road ⇒ (3m/4m)

Banking of roads : The phenomenon in which the edges are raised for the curved roads above the inner edge to provide the necessary centripetal force to the vehicles so that they take a safe turn.

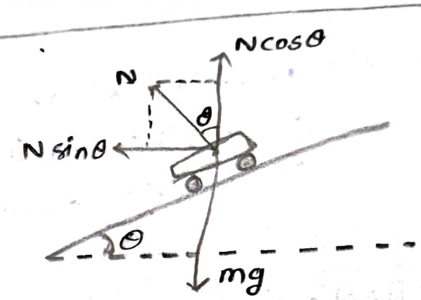


Fig: vehicle on a banked road.

$N\cos\theta = mg$ —— ①

$N\sin\theta = \frac{mv^2}{r}$ —— ②

Divide equation ② by ①

$$\frac{\frac{\cancel{m}v^2}{r}}{\cancel{m}g} = \frac{\cancel{N}\sin\theta}{\cancel{N}\cos\theta}$$

$\frac{v^2}{rg} = \tan\theta$ —— ③

$v^2 = rg\tan\theta$

$\boxed{V_s = \sqrt{rg\tan\theta}}$ —— ④

From equation ③

$$\frac{v^2}{rg} = \tan\theta$$

$$\boxed{\theta = \tan^{-1}\left(\frac{v^2}{rg}\right)}$$

where, θ = banking angle

ⓐ **Most safe speed :→**

$$\boxed{V_s = \sqrt{rg\tan\theta}}$$

ⓑ **Banking angle :→**

$$\boxed{\theta = \tan^{-1}\left(\frac{v^2}{rg}\right)}$$

ⓒ **Sped limit**

(2m) * **Banked road : Lower speed limit**

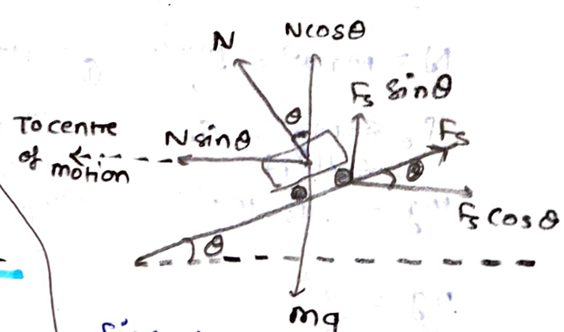


Fig: lower speed limit.

$mg = N\cos\theta + F_s\sin\theta$ —— ①

$\frac{mv^2}{r} = N\sin\theta - F_s\cos\theta$ —— ②

② by ① and also devide by $N\cos\theta$, we get

$$\frac{v^2}{rg} = \frac{\tan\theta - \mu_s}{1 + \mu_s\tan\theta}$$

$$v^2 = rg\left(\frac{\tan\theta - \mu_s}{1 + \mu_s\tan\theta}\right)$$

$$\boxed{V_{min} = \sqrt{rg\left(\frac{\tan\theta \pm \mu_s}{1 + \mu_s\tan\theta}\right)}}$$

* Banked road : Upper speed limit

$$mg + f_s \sin\theta = N\cos\theta$$
$$mg = N\cos\theta - f_s \sin\theta \quad \text{--- (1)}$$
$$\frac{mv^2}{r} = N\sin\theta + f_s\cos\theta \quad \text{--- (2)}$$

(2) by (1) and also divide by $N\cos\theta$, we get

$$\frac{v^2}{rg} = \frac{\tan\theta + \mu_s}{1 - \mu_s\tan\theta}$$
$$v^2 = rg\left(\frac{\tan\theta + \mu_s}{1 - \mu_s\tan\theta}\right)$$
$$\boxed{V_{max} = \sqrt{rg\left(\frac{\tan\theta + \mu_s}{1 - \mu_s\tan\theta}\right)}}$$

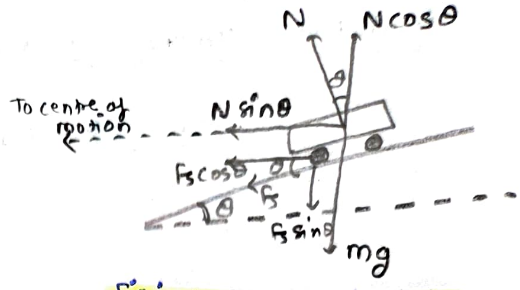


Fig : upper speed limit

Conclusion

$$V_{min} = \sqrt{rg\left(\frac{\tan\theta - \mu_s}{1 + \mu_s\tan\theta}\right)}$$
$$V_{max} = \sqrt{rg\left(\frac{\tan\theta + \mu_s}{1 - \mu_s\tan\theta}\right)}$$

when, $\mu_s = 0$

$$\boxed{V_s = \sqrt{rg\tan\theta}}$$

Imp

$\boxed{\sin 37^\circ = \frac{3}{5}}$ $\boxed{\cos 37^\circ = \frac{4}{5}}$ $\boxed{\tan 37^\circ = \frac{3}{4}}$

Imp (4) **Conical pendulum** → [3m/4m]

Define → string moves along the surface of a right circular cone of vertical axis and the point object performs a (practically) uniform horizontal circular motion. In such a case the system is called a conical pendulum.

$$T_0\cos\theta = mg \quad \text{--- (1)}$$
$$T_0\sin\theta = mr\omega^2 \quad \text{--- (2)}$$

(2) by (1)

$$\frac{\cancel{m}r\omega^2}{\cancel{m}g} = \frac{\cancel{T_0}\sin\theta}{\cancel{T_0}\cos\theta}$$
$$\frac{r\omega^2}{g} = \tan\theta$$
$$\boxed{\omega = \sqrt{\frac{g\tan\theta}{r}}}$$

period (T) : Time taken by bob to complete one revolution called as period.

$$\boxed{\therefore T = \frac{2\pi}{\omega}}$$
$$\boxed{T = \frac{2\pi}{\sqrt{\frac{g\tan\theta}{r}}}}$$

From △CBA

$$\sin\theta = \frac{r}{L}$$
$$r = L\sin\theta$$
$$\therefore T = \frac{2\pi}{\sqrt{\frac{g\,\cancel{\sin\theta}}{\cos\theta \, L\,\cancel{\sin\theta}}}}$$
$$T = \frac{2\pi}{\sqrt{g/L\cos\theta}}$$
$$\boxed{T = 2\pi\sqrt{\frac{L\cos\theta}{g}}}$$

period

Reciprocal of time period.

$$\text{Frequency}(n) = \frac{1}{T}$$
$$\boxed{n = \frac{1}{2\pi}\sqrt{\frac{g}{L\cos\theta}}}$$

where,
T_0 = Tension
T = period
L = length of the string

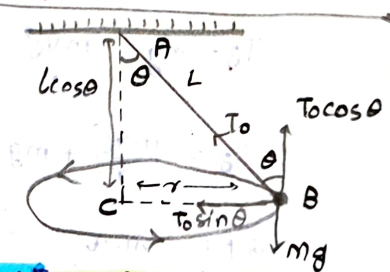


Fig: In an inertial frame

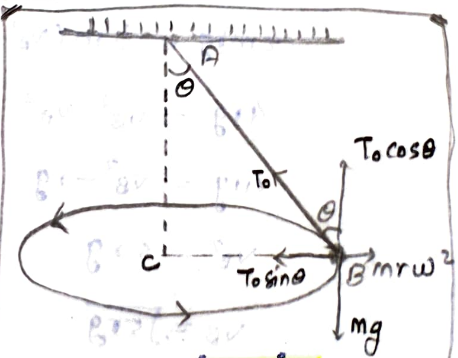


Fig In a non-inertial frame

Vertical circular motion [2m]

Imp * Point mass undergoing vertical circular motion Under gravity :

Case-I :- Mass tied to a string

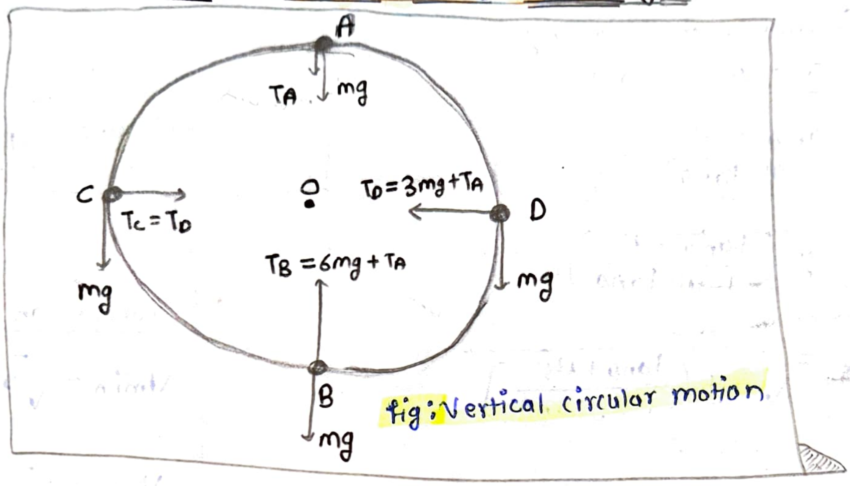


fig : Vertical circular motion

Uppermost position (A) ⇒

$$T_A + mg = \frac{mv_A^2}{r} \quad \text{—— (1)}$$

$$\boxed{T_A = \frac{mv_A^2}{r} - mg}$$

If, $\boxed{T_A = 0}$

$$0 + \cancel{m}g = \frac{\cancel{m}v_A^2}{r}$$

$$v_A^2 = rg$$

minimum speed of uppermost position.

$\boxed{V_A = \sqrt{rg}}$ → highest position.

Imp:-
Tension (CPF) → Direction
weight (mg) ⇒ speed change

Lowermost position (B) ⇒

$$T_B - mg = \frac{mv_B^2}{r}$$

$$\boxed{T_B = \frac{mv_B^2}{r} + mg}$$

G.P.E = ΔK.E

$$mgh = \frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2$$

$$2\cancel{m}rg = \frac{1}{2}\cancel{m}(v_B^2 - v_A^2)$$

$$\boxed{4rg = v_B^2 - v_A^2}$$

$$4rg = v_B^2 - rg$$

$$v_B^2 = 5rg$$

$$\boxed{V_B = \sqrt{5rg}}$$

Now,

$$T_B - T_A = \frac{mv_B^2}{r} + mg - \frac{mv_A^2}{r} + mg$$

$$T_B - T_A = \frac{m}{r}(5rg - rg) + 2mg$$

$$T_B - T_A = \frac{m}{\cancel{r}} \times 4\cancel{r}g + 2mg$$

$$T_B - T_A = 4mg + 2mg$$

$$\boxed{T_B - T_A = 6mg}$$

ऊपर से नीचे आते वक्त PE convert होती KE में.

KE नीचे ज्यादा ~~कम~~ है than ऊपर

energy conservation होती है।

Position when the string is horizontal (C and D) ⇒

$$\boxed{T_C - T_A = T_D - T_A = 3mg}$$

$$\boxed{(V_C)_{min} = (V_D)_{min} = \sqrt{3rg}}$$

Imp:- mass tied to a string.

$$T_{any\ position} = \frac{mV_P^2}{r} + mg\cos\theta$$

$$V_{any\ position} = \sqrt{rg(3+2\cos\theta)}$$

Case - II :- Mass tied to a rod

lowermost position.

$$\boxed{V_B = \sqrt{4rg}}$$

$$\boxed{T_B - T_A = 6mg}$$

(V_{min}) at the rod horizontal position $= \sqrt{2rg}$

* Sphere of Death : (मृत्यु गोला)

- Force due to tension T is replaced by the normal reaction force N.

* Vehicle at the top of a convex over-bridge : 2m/3m

$$mg - N = \frac{mv^2}{r}$$

If $N = 0$

$$\cancel{m}g = \frac{\cancel{m}v^2}{r}$$

$$v^2 = rg$$

$$\boxed{V_{max} = \sqrt{rg}}$$

As the speed is ↑, N goes on decreasing. Normal reaction is an indication of contact.

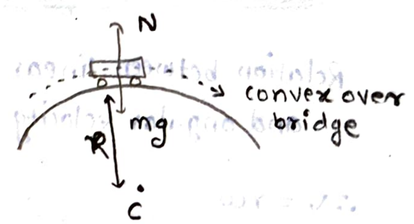


Fig:- Vehicle on a convex over bridge

Q. Define moment of inertia gives its SI unit and dimension?

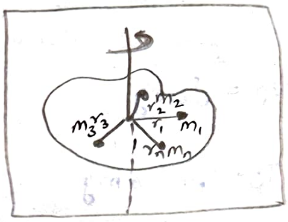
Imp

Moment of Inertia

2m|3m

Statement ⇒ Moment of inertia is defined as the quantity expressed by the body resisting angular acceleration which is the sum of the product of the mass of every particle with its square of a distance from the axis of rotation.

$$\boxed{I = MR^2}$$



$\therefore I = m_1 r_1^2 + m_2 r_2^2 + \dots m_n r_n^2$

$$\boxed{I = \sum_{i=1}^{n} M_i r_i^2}$$

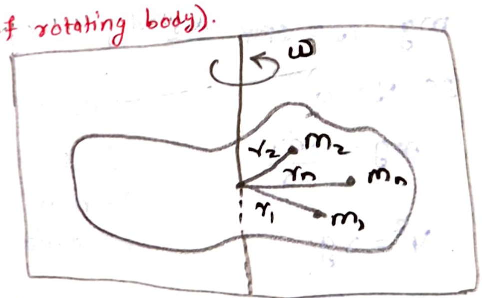
SI unit :- $kg\,m^2$

Dimension :- $[M^1 L^2 T^0]$

	Linear	angular
	s	θ
	v	ω
	a	α
mass	m	I
Force	$F = ma$	$\tau = I\alpha$ → Torque
momentum	$P = mv$	$L = I\omega$
kinetic energy	$KE = \frac{1}{2}mv^2$	$(KE)_{rota.} = \frac{1}{2}I\omega^2$

3m Imp

* K.E. of system of N particles :- (KE of rotating body).



Relation between linear velocity and angular velocity

$\therefore v = r\omega$

⇒ KE of First particle

$E_1 = \frac{1}{2} m_1 v_1^2$

$E_1 = \frac{1}{2} m_1 r_1^2 \omega^2$

⇒ KE of 2nd particle

$E_2 = \frac{1}{2} m_2 r_2^2 \omega^2$

⇒ KE of nth particles

$E_n = \frac{1}{2} M_n r_n^2 \omega^2$

Now,

→ Total K.E. Energy of body,

$E = E_1 + E_2 + \dots + E_n$

$E = \frac{1}{2} M_1 r_1^2 \omega^2 + \frac{1}{2} M_2 r_2^2 \omega^2 + \dots + \frac{1}{2} m_n r_n^2 \omega^2$

$E = \frac{1}{2} [M_1 r_1^2 + m_2 r_2^2 + \dots + M_n r_n^2] \omega^2$

we know,

$I = m_1 r_1^2 + m_2 r_2^2 + \dots + m_n r_n^2$

$$\boxed{\therefore E = \frac{1}{2} I \omega^2}$$

where, $\omega = 2\pi n$

$\omega = \frac{2\pi}{T}$

↳ k.E. of rotating body

SI unit ⇒ Joule (J)

Dimension ⇒ $[M^1 L^2 T^{-2}]$

Consider a particle of masses, $m_1, m_2, \dots, m_n$ and situated distances, $r_1, r_2, \dots, r_n$ resp.

* **Moment of inertia of a uniform ring :-**

$$\boxed{I_o = MR^2}$$

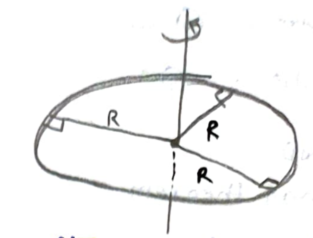


Fig: Moment of inertia of a ring.

* **Moment of inertia of a uniform Disc :-**

surface density is,

$$\sigma = \frac{m}{A} = \frac{mass}{Area}$$

$$\sigma = \frac{M}{\pi R^2}$$

Small element.

$A = 2\pi r\, dr$ $\quad \sigma = \frac{dM}{dA}$

$$\therefore \sigma = \frac{dM}{2\pi r\, dr}$$

$$\boxed{dM = 2\pi\sigma\, r\, dr}$$

now,

$$I_o = \int_0^R dI = \int_0^R dM r^2$$

$$I = \int_0^R 2\pi\sigma r\, dr\, r^2$$

$$I = 2\pi\sigma \int_0^R r^3 dr$$

$$I = 2\pi\sigma \left[\frac{r^4}{4}\right]_0^R$$

$$I = \frac{2\pi\sigma}{4}\left[R^4 - 0^4\right]$$

$$I = \frac{2\cancel{\pi} M}{4 \times \cancel{\pi} \cancel{R^2}} R^{\cancel{4}\,2}$$

$$\boxed{I_o = \frac{MR^2}{2}}$$

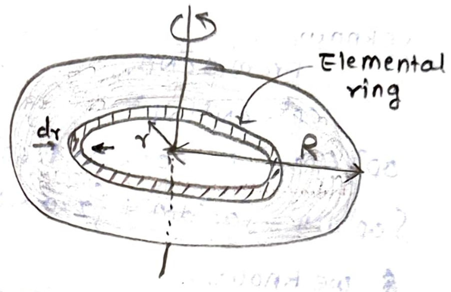


Fig: Moment of inertia of a disk.

Instead of radius
Diameter [हेता ती]

$$\boxed{I_d = \frac{MR^2}{4}}$$

Imp **Radius of Gyration** [2m]

statement : The distance from the axis of rotation to a point where total mass of any body is ~~SP~~ supposed to be concentrated, so that the moment of inertia about the axis may remain same.

defn:

- It is actually perpendicular distance from point mass to the axis of rotation.

$$I = mk^2$$

$$\boxed{K = \sqrt{\frac{I}{M}}}$$

SI unit: metre (m)

Dimension:- $[M^0 L^1 T^0]$

physical significance :

- If the particles of a body are distributed close to the axis, the MI is less, and hence K is less.
- If particles are distributed away from the axis of rotation, the MI more, hence K is more.

Thus K is a measure of distribution of mass of a body about the given axis of rotation.

Imp

Theorem of parallel axes [3m|4m]

$I_c = \int CD^2\, dM$ —— ①

$I_o = \int OD^2\, dM$ —— ②

From $\triangle$ OND,

by pythagoras theorem,

$OD^2 = ON^2 + ND^2$

$OD^2 = (OC + CN)^2 + ND^2$

$OD^2 = OC^2 + 2 \times OC \times CN + \underline{CN^2 + ND^2}$

we know,

From $\triangle$ CND,

$\boxed{CD^2 = CN^2 + ND^2}$

$OD^2 = OC^2 + 2 \times OC \times CN + CD^2$

taking integration on both side.

$\int OD^2 dM = \int OC^2 dM + \int 2 \times OC \times CN\, dM + \int CD^2 dM$

we know,

$\int OD^2 dM = I_o$, $OC = h$, $\int CD^2 dM = I_c$

$I_o = \int h^2 dM + 2\int h\, CN\, dM + I_c$

$I_o = h^2 \int dM + 2h \int CN\, dM + I_c$

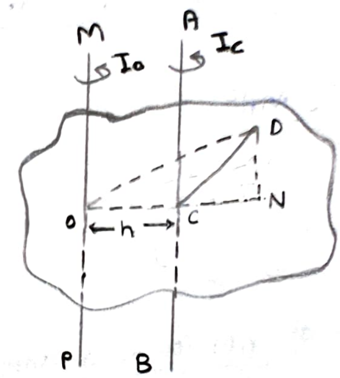


fig:- Theorem of parallel axes.

we know,

$\int dM = M$ and $\int CN\, dM = 0$

$I_o = Mh^2 + 2h \times 0 + I_c$

$$\boxed{I_o = I_c + Mh^2}$$

where,

I_o = MI of body an axis.

I_c = MI of body passing through centre.

h = distance betn two parallel axes.

Statement ⇒ The moment of inertia of an object about any axis is the sum of its moment of inertia about an axis parallel to the given axis, and passing through the centre of mass and the product of the mass of the object and the square of the distance betn the two axes.

Imp

Theorem of perpendicular axes [3m|4m]

$I_x = \int y^2 dM$ —— ①

$I_y = \int x^2 dM$ —— ②

$I_z = \int z^2 dM$ —— ③

Adding eqn ① & ②

$I_x + I_y = \int y^2 dM + \int x^2 dM$

$I_x + I_y = \int (x^2 + y^2)\, dM$

$I_x + I_y = \int z^2 dM$ ---- [From $\triangle$ ONP]

$I_x + I_y = I_z$ ---- From ③

$$\boxed{I_z = I_x + I_y}$$

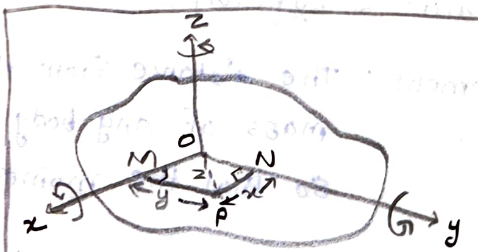


fig: Theorem of ⊥r axes.

Statement ⇒ The moment of inertia of a laminar object about an axis ⊥r to its plane is the sum of its moment of inertia about two mutually ⊥r axes in its plane, all the three axes being concurrent.

Angular momentum OR Moment of Linear momentum

2m

Statement ⇒ The property of any rotating object given by moment of inertia times angular velocity.

$\boxed{L = I\omega}$ $\boxed{\vec{L} = \vec{r} \times \vec{p}}$ $\boxed{L = rp\sin\theta}$ → cross product.

MI of body

angular velocity.

Relation between linear momentum and angular momentum.

SI unit ⇒ m kg m/s ⇒ $kg\,m^2/s$

Dimension ⇒ $[M^1 L^2 T^{-1}]$

* Expression For Angular momentum in Terms of Moment of Inertia :-

3m/4m

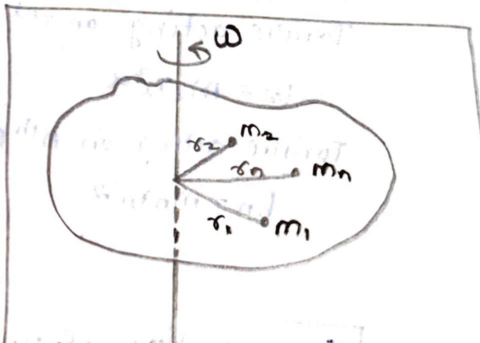


fig : A body of N particles.

Relation between linear velocity and angular velocity,

$\therefore v = r\omega$

$p = mv \quad \therefore p = mr\omega$

linear momentum.

Angular momentum of 1st particle

$\therefore L_1 = r_1 m_1 r_1 \omega$ ----- $[\because L = pr]$ → Relation between linear momentum and angular momentum.

$\therefore L_1 = m_1 r_1^2 \omega$

Angular momentum of 2nd particle

$\therefore L_2 = m_2 r_2^2 \omega$

now,

Angular momentum of nth particle.

$\therefore L_n = m_n r_n^2 \omega$

Total angular momentum of a body is,

$L = L_1 + L_2 + \cdots + L_n$

$L = m_1 r_1^2 \omega + m_2 r_2^2 \omega + \cdots + m_n r_n^2 \omega$

$L = [m_1 r_1^2 + m_2 r_2^2 + \cdots + m_n r_n^2]\,\omega$

since,

$I = m_1 r_1^2 + m_2 r_2^2 + \cdots + m_n r_n^2$

$\boxed{\therefore L = I\omega}$

⇒ Angular momentum in terms of moment of inertia.

Amount of force   position of force   Angle.
depends on.

$\boxed{\tau = I\alpha}$

Imp

Expression For Torque in Terms of Moment of Inertia

$\boxed{3m}$

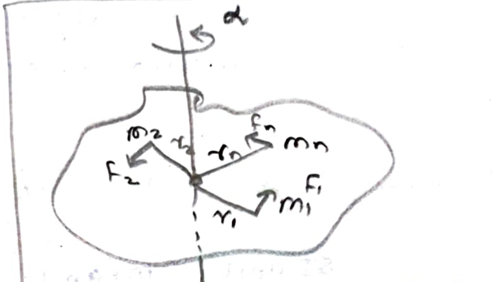


fig:- Rigid rotating object

Relation between linear acceleration and angular acceleration

$a = r\alpha$

Force acting on a particle,

$F = ma \quad \therefore F = mr\alpha$

$\therefore \tau = rF$

Torque acting on a 1st particle

$\tau_1 = F_1 r_1 \quad \therefore \tau_1 = m_1 r_1 \alpha r_1$

$\tau_1 = m_1 r_1^2 \alpha$

→ Torque acting on 2nd particle

$\tau_2 = m_2 r_2^2 \alpha$

→ Torque acting on nth particle

$\tau_n = m_n r_n^2 \alpha$

Total Torque acting on a body,

$\tau = \tau_1 + \tau_2 + \dots + \tau_n$

$\tau = m_1 r_1^2 \alpha + m_2 r_2^2 \alpha + \dots + m_n r_n^2 \alpha$

$\tau = [m_1 r_1^2 + m_2 r_2^2 + \dots + m_n r_n^2]\,\alpha$

$\boxed{\tau = I\alpha}$

Imp.

Conservation of Angular Momentum

$\boxed{2m/3m}$ State.

$\vec{L} = \vec{r} \times \vec{P}$ → Angular momentum

$\frac{d\vec{L}}{dt} = \frac{d}{dt}(\vec{r} \times \vec{P})$

$\frac{d\vec{L}}{dt} = \vec{r} \times \frac{d\vec{P}}{dt} + \frac{d\vec{r}}{dt} \times \vec{P}$

Velocity.

newtons law

but, $\boxed{\vec{F} = \frac{d\vec{P}}{dt}}$ and $\boxed{\frac{d\vec{r}}{dt} = \vec{v}}$

$\frac{d\vec{L}}{dt} = \vec{r} \times \vec{F} + \vec{v} \times (m\vec{v})$

$\frac{d\vec{L}}{dt} = \vec{r} \times \vec{F} + m(\vec{v} \times \vec{v})$

we know,

$\vec{r} \times \vec{F} = \vec{\tau}$

$\boxed{\because \vec{v} \times \vec{v} = vv\sin 0 = 0}$

$\boxed{\frac{d\vec{L}}{dt} = \vec{\tau} + 0}$ angular momentum

$\boxed{\vec{\tau} = \frac{d\vec{L}}{dt} = 0}$ $\quad \therefore \vec{L} = \text{constant}$

$\boxed{\vec{\tau} = 0}$ $\quad \frac{d\vec{L}}{dt} = 0$

~~Statement:~~

Principle:- If no resultant external torque acts on a rotating body, then its angular momentum is conserved.

Rolling motion

Define: A motion is said to be rolling motion if a body possess both linear motion and rotational motion.

Rolling motion = translation motion + rotating motion.

⇒ $(KE)_{Rolling} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

⇒ $(KE)_{Rolling} = \frac{1}{2}mv^2\left(1+\frac{k^2}{R^2}\right)$

⇒ $(K.E)_{Rolling} = \frac{1}{2}m\omega^2(R^2+k^2)$

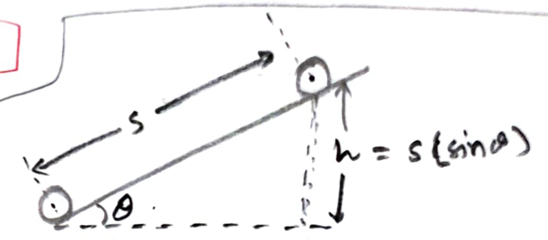


fig: Rolling along an incline

* Linear acceleration and speed while pure rolling Down an inclined plane :

$PE = (KE)_{rolling}$

$\cancel{m}gh = \frac{1}{2}\cancel{m}v^2\left(1+\frac{k^2}{R^2}\right)$

$\frac{2gh}{1+\frac{k^2}{R^2}} = v^2$

⇒ $\boxed{v = \sqrt{\frac{2gh}{\left(1+\frac{k^2}{R^2}\right)}}}$ → Velocity.

we know,

$2as = v^2 - u^2$ ---- $[v^2 = u^2 + 2as]$ → 3rd kinematical eqn.

$\cancel{2}a \times \frac{\cancel{h}}{\sin\theta} = \frac{\cancel{2}g\cancel{h}}{\left(1+\frac{k^2}{R^2}\right)}$ $- 0$ ---$\left[s = \frac{h}{\sin\theta}\right]$

u → initial velocity.
v → Final velocity.
s → Displacement.

$\boxed{a = \frac{g\sin\theta}{\left(1+\frac{k^2}{R^2}\right)}}$ → acceleration.

Imp

Centripetal force	Centrifugal force.
- It is always directed towards the centre of circle along the radius.	- It is always directed away from the centre of circle along the radius.
- It arises in an inertial Frame of reference.	- It arises in an non-inertial Frame of reference.
- It is a real force.	- It is an imaginary force.

$v = u + at$
$s = ut + \frac{1}{2}at^2$
$v^2 = u^2 + 2as$

} kinematical equations.